1. Photometry and spectroscopy of Nova Del 2013 **(60 p)**

Classical nova V339 Del (Nova Delphini 2013) was discovered by Koichi Itagaki at 6.8 magnitude on 14 August 2013 at 14:01 UT (MJD = 56518.584). Both professional and amateur astronomers analysed the photometry and spectroscopy of the nova. Less than 10 hours after the alert, when the night falls at the Piszkéstető Mountain Station of the Konkoly Observatory of the Hungarian Academy of Sciences, Hungarian astronomers took the first spectrum data of the nova using the eShel echelle spectrograph in the Gothard Astrophysical Observatory of Loránd Eötvös University attached to the 1 meter telescope of the Konkoly Observatory.

Refer to Fig 1.1 and Fig 1.2 to complete the questions. The larger versions of Fig 1.1 and Fig 1.2 are found on separate A3 papers.

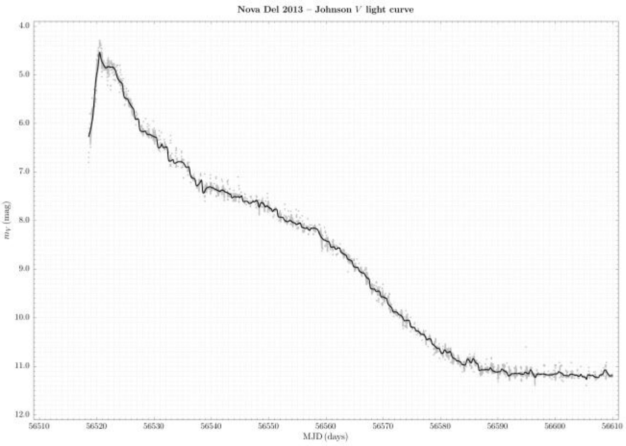

Fig. 1.1: Nova Del 2013 – Johnson V light curve

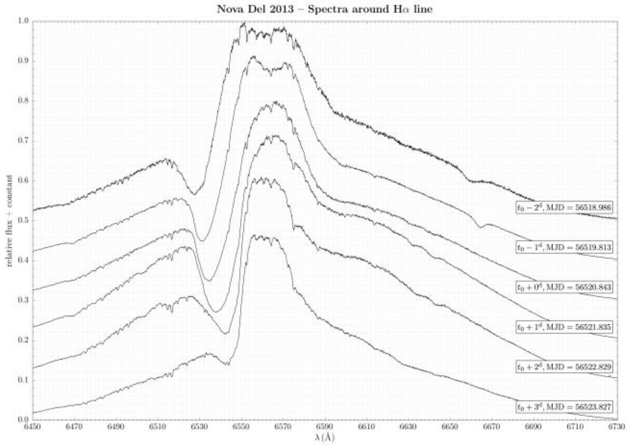

Fig. 1.2: Nova Del 2013 – Spectra around Hα line

Fig. 1.1 shows the visual light curve of the nova based on the data downloaded from the website of AAVSO (American Association of Variable Star Observers). On the horizontal and vertical axes, the Modified Julian Date (MJD = JD−2 400 000.5) of the observations and the Johnson V magnitudes are plotted, respectively. The grey circles (about 38000 data points) represent the measured values, while the continuous black line is the result of smoothing the data with a Gaussian filter (Full Width at Half Maximum = 0.5 day) to define an "average" light curve from the data points.

The rate of decline can be characterized by the values t_2 and t_3, which show the time interval in days in which a nova fades from its maximum brightness by 2 and 3 magnitudes.

A few empirical formulae between the peak of the absolute magnitude in the *V* band (M_0) and t_2, t_3 values can be found in the following literature:

(a) $M_0 = -7.92 - 0.81 \arctan \frac{1.32 - \log t_2}{0.23}$ (Della Valle, M. & Livio, M.: 1995, *ApJ* **452**, 704)

(b) $M_0 = -11.32 + 2.55 \log t_2$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

(c) $M_0 = -11.99 + 2.54 \log t_3$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

The $E(B-V)$ color excess of Nova Del 2013 (Chochol, D. et al.: 2014, *Contrib. Astron. Obs. Skalnaté Pleso* **43**, 330) is:

$$E(B-V) = 0.184 \pm 0.035$$

Fig 1.2 shows the nova spectra taken in the wavelength region around the Hα line on six consecutive nights before and after the time of the maximum brightness (t_0). The individual spectra have been shifted vertically for clarity. The Modified Julian Dates (MJD) of the observations are listed on the right hand side of each spectrum slice.

The Hα line shows the so-called P Cygni profile with very broad wings, which is typical not of novae only, but is present in almost all spectral types and is a reliable sign of a massive radial motion of matter ejected from the star. The P Cygni profile is composed of a strong, broad emission peak which is considered to be centered at the rest wavelength in air λ_0 of the line – for Hα $\lambda_0 = 6562.82$ Å – and a usually weaker, blueshifted absorption component. The expansion (radial) velocity of the shell can be approximated from the measured wavelength λ of the absorption peak using the well known Doppler formula connecting the displacement $\Delta\lambda = \lambda - \lambda_0$, the radial velocity v_r, and c, the speed of light.

Assume that the Hα line showing P Cygni profile is excited in the outermost part of the spherically expanding shell, and its extent at the moment of taking the first spectrum was still negligible.

a) From Fig. 1.1, estimate the Modified Julian Date of the peak magnitude (MJD_0) and the value of the peak magnitude itself. Consider the error of this brightness value to be 0.05^m. (3 p)

b) Estimate the Modified Julian Dates based on the time interval (days) in which the nova has faded by 2 and 3 magnitudes, then calculate t_2 and t_3 values. (6 p)

c) With reference to t_2 and t_3 from (b), determine the peak absolute magnitude of the nova using all three empirical formulae listed earlier, calculate their mean (M_0) and their standard deviation, and consider this latter as the uncertainty of M_0. (5 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the value of the color excess $E(B-V)$, determine the interstellar extinction A_V and its uncertainty in the direction of the nova. Use $R_V = 3.1$, without error. (4 p)

e) Estimate the distance to the nova and its uncertainty. Give the result in kpc. (11 p)

f) Measure the central wavelengths of the P Cygni absorption features plotted in Fig. 1.2 (refer to magnified version), and calculate the corresponding radial velocities. No error estimation is needed. (14 p)

g) Plot these radial velocities against the Modified Julian Dates of the observations. (6 p)

h) From the graph in (g), estimate the physical radius of the envelope at the end of the time interval. Give the answer in astronomical units (au). (7 p)

i) Knowing the distance to the nova and the physical radius of the spherical envelope 5 days after the discovery, estimate the apparent angular diameter of the envelope then. (4 p)

2. Triply eclipsing hierarchical triple stellar system (90 p)

HD 181068 was one of the brightest targets which was continuously observed during the almost 4-year-long primary mission of NASA's exoplanet-hunter *Kepler* space telescope. The spacecraft observed $\approx 3-4\times10^{-3}$ magnitude dimmings every 0.453 days. (Note: The even dimmings were slightly smaller amplitude than the odd ones.) Furthermore, additional 0.007 magnitude, 2.3-day-long dimmings were detected every 22.7 days.

The correct explanation of this very unusual photometric behaviour was given by Hungarian astronomers. They found that HD 181068 is a compact hierarchical triple stellar system seen almost edge-on.

Hierarchical triple star systems consist of three stars; A, B, and C. Two of these stars (B and C) form an inner or close stellar binary system, whilst the outer component (star A) orbits at a distance from the inner system significantly larger (usually orders of magnitude) than the semi-major axis of the inner system. The schematic view of an example of a triple star system is illustrated in Fig. 2.1.

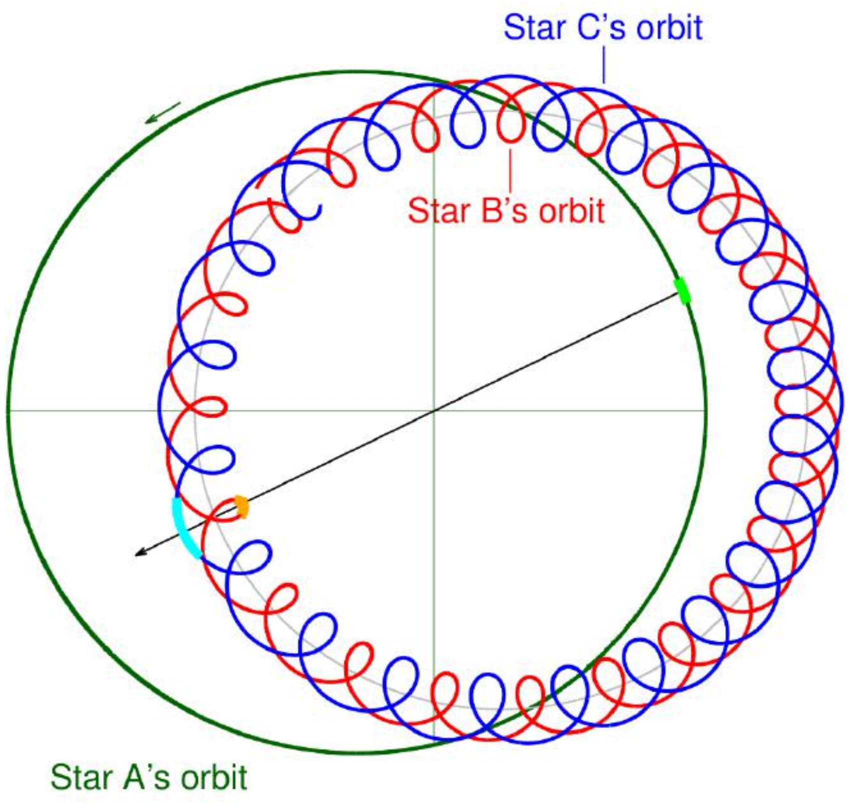

Fig. 2.1: The schematic pole-on view of a hypothetical , hierarchical triple stellar system. The black arrow is directed towards the Earth. The thick segments of the three orbits represent the stars' orbital arcs during an outer eclipse.

Mathematically, the motion of a hierarchical triple system can be well approximated with two unperturbed Keplerian two-body motions; (1) Keplerian motion of the inner binary. (2) The centre of mass of this close binary and the third star revolves on a second Keplerian orbit, "outer binary".

In this problem, stars B and C form a $P_1 = 0.9056768$-day-period eclipsing binary, while the centre of mass of these stars with star A forms the $P_2 = 45.4711$-day-period outer binary. As the orbital plane of this outer orbit is seen almost edge-on from the Kepler spacecraft (and from the Earth), during their revolution on the outer orbit, stars B and C eclipse not only each other, but also star A or, a half outer revolution later are eclipsed by it, causing the extra dimmings.

i. *Determination of the physical stellar sizes (and other quantities) from the geometry of the eclipses*

These assumptions are used throughout this section: (1) both the inner and outer orbits are exactly circular, (2) the orbital planes of the inner and outer orbits are identical, and (3) this plane is seen exactly edge-on (i.e. $i_1 = i_2 = 90°$ and $i_{\mathrm{rel}} = 0°$). Let's consider the extra dimmings, which are central eclipses (i.e. either occultations or transits – annular eclipses), therefore, these events have four contacts. In the case of an ordinary eclipsing binary (or of a transiting exoplanet) at the times of the outer contacts the sky-projected disks of the two objects connect with each other at one point from outside, while at the inner contacts they approach each other from inside. While this last statement is also valid for outer eclipses, the situation becomes more complex, because instead of two, three stars are involved into the eclipses. However, despite this fact, we can certainly define the times of each contact from the light curve, and furthermore, we can also decide explicitly which member of the inner binary is involved in a given contact. (The other member, of course, is always star A.)

In the table below, the accurate times of some contacts of different eclipses observed by the Kepler spacecraft, the contact types and the stars are documented. Time is expressed in barycentric Julian Days (BJD).

event no.	contact	stars	BJD	φ_1	φ_2
1	I	A, B	2455476.1096		
	II	A, C	2455476.4245		
	III	A, B	2455477.9677		
	IV	A, B	2455478.4722		
2	I	A, B	2455521.5217		
3	III	A, C	2455568.9434		
4	I	A, C	2455612.4733		
	III	A, C	2455614.3571		
5	III	A, B	2455659.9241		
	IV	A, C	2455660.2422		

a) Given that $T_{01} = 2455051.2361$ and $T_{02} = 2455522.7318$ denote the time of an inferior conjunction of the inner and outer binaries respectively (i.e. that time, when, from the perspective of the observer, star C eclipses star B, and when star A eclipses the centre of mass of stars B and C.)

Define

$\varphi_1(t) = \{(t - T_{01})/P_1\}$, and $\varphi_2(t) = \{(t - T_{02})/P_2\}$

as the photometric phases of the inner and outer binaries respectively. $\{x\}$ denotes the decimal part of the real number x. If $\{x\} < 0$, use $\{x\} + 1$ instead. Calculate the phases for the times of the tabulated contact events and **write the answers in the appropriate columns of the table on the answer sheet**. Round your answers to four decimal places. (10 p)

b) Determine, whether star A, or the close binary (i.e. stars B and C) were closer to the observer during each eclipsing event. **Write your answer in the table on the answer sheet.** (5 p)

c) Using the table above, calculate (1) the dimensionless radius of each star relative to the semi-major axis of the outer orbit ($R_{\mathrm{A,B,C}}/a_2$), (2) the ratio of the semi-major axes of the two orbits (a_1/a_2) and (3) the mass ratio of stars B and C ($q_1 = m_\mathrm{C}/m_\mathrm{B}$). *Hint:* Use at least four decimal place accuracy in your calculations. Be cautious, it may not be possible to use all theoretically possible contact combinations with a given limited accuracy of time data. (30 p)

d) Based on the results obtained above, calculate the outer mass ratio ($q_2 = m_\mathrm{BC}/m_\mathrm{A}$). (8 p)

ii. *Dynamical determination of the stellar masses using radial velocity (RV) and eclipse timing variations (ETV) measurements*

To obtain RV data, ground-based spectroscopic follow up observations were carried out with four different instruments. Only the lines of star A were detectable in all spectra. Plotting all the measurements against time, the RV curve was nicely fitted in the following form:

$$V_\mathrm{rad,A} = V_\gamma + K_\mathrm{A} \sin \phi_\mathrm{RV},$$

where V_γ is the systemic velocity and K_A is the velocity amplitude:

$$V_\gamma = 6.993 \pm 0.011 \text{ km s}^{-1},\ K_\mathrm{A} = 37.195 \pm 0.053 \text{ km s}^{-1},$$

$$P_2 = 45.4711 \pm 0.0002 \text{ d},\ \phi_\mathrm{RV} = \frac{2\pi}{P_2}[t - (2455522.7318 \pm 0.0095)].$$

Furthermore, the researchers determined the mid-times of the regular eclipses of the close binary (formed by stars B and C), and found that the occurrence of for example the eclipsing minima belonging to the N^th orbital revolution can be described by the simple expression:

$$T_N = T_0 + P_1 N + A_\mathrm{ETV} \sin\left(\frac{2\pi}{P_2} P_1 N + \phi_0\right),$$

where

$$T_0 = \text{BJD } 2455051.23607 \pm 5 \times 10^{-5},\ P_1 = 0.9056768 \pm 3 \times 10^{-7} \text{ d},$$

$$A_\mathrm{ETV} = 0.001446 \pm 0.000110 \text{ d},\ \phi_0 = -0.76779 \pm 0.01937 \text{ rad}.$$

In this expression A_ETV is the amplitude of the eclipse timing variation, T_0 denotes the mid-eclipse time of the reference (zeroth) primary eclipse, and N is the cycle number, which is an integer for primary eclipses (i.e. when the slightly fainter star C eclipses star B), and half-integer for secondary ones (i.e. when star B eclipses star C).

Determine (1) again the mass ratio ($q_2 = m_\mathrm{BC}/m_\mathrm{A}$) of the centre of mass of the inner binary and star A using only the results obtained in point ii., (2) the mass of component A (m_A) and (3) the total mass of the inner, close binary (m_BC). Calculate the errors for (1), (2), and (3) in masses. *Hint:* You can save much time by expressing the masses in solar mass and the orbital separations either in solar radius or au. (22 p)

iii. Using results obtained in questions 1 and 2, determine the masses of stars B and C respectively and calculate the physical dimensions of all three stars (i.e. stellar radii in physical units). (15 p)